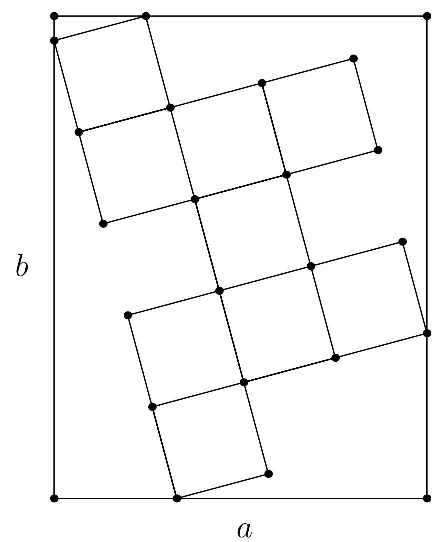
UTMC Junior Team Round

1. (4) One day, Charles played around with the letters of his favourite word, $ROSEBUD$. In how many ways could he arrange the letters of $ROSEBUD$ such that:
   - the letter R appears somewhere before the letter B, and
   - no two vowels are adjacent to each other?
2. (5) David is playing a casino game. Each round, he has a $\frac{1}{3}$ chance of winning a jackpot revenue of 2000 tickets. If he does not win this jackpot, he wins a revenue of either 5 tickets or 15 tickets, with an equal chance of winning each prize. If David plays exactly 3 times, and the number of tickets he owns does not change between games, compute the probability that his total revenue is at least 2020 tickets.
3. (6) Find all nonnegative integers n such that removing the last digit of $(3n+2)^2$ results in the number n^2.
4. (7) The figure below consists of 9 congruent squares inscribed within a larger rectangle. If $a = 126$ and $b = 159$, compute the area of one of the 9 congruent squares.



5. (7) We have 3 prime numbers p, q, and r such that $p + q + r = 50$ and $pq + qr + rp$ is the maximum possible for all such triplets of primes. Find pqr.
6. (8) A tower with length and width of 2 and height of 100 is constructed with 1 by 1 by 1 blocks. Then, a colour of either red or blue assigned at random, with each block being colored 1 color. What is the expected number of adjacent pairs of blocks with opposite colours?
7. (9) The sequence (x_n) is defined by $x_0 = 1$ and $x_{n+1} = 2022^n + x_n$ for all $n \geq 0$. Compute the last two digits of x_{2020}.

8. (9) Let triangle $\Delta A_0B_0C_0$ have an area of 400. For all integers $k > 0$, construct $\Delta A_kB_kC_k$ recursively as follows: let A_k, B_k, and C_k be points on sides $B_{k-1}C_{k-1}$, $C_{k-1}A_{k-1}$, and $A_{k-1}B_{k-1}$, respectively, such that:

$$\frac{A_kB_{k-1}}{A_kC_{k-1}} = \frac{B_kC_{k-1}}{B_kA_{k-1}} = \frac{C_kA_{k-1}}{C_kB_{k-1}} = \frac{1}{6}$$

Compute the total area covered by all the points that are inside an odd number of these triangles.

9. (10) Let ABC be a triangle with $AB = 7$, $BC = 4$, and $CA = 9$. Let M be the midpoint of BC. Let the internal angle bisector of $\angle AMB$ intersect AB at D, and let the internal angle bisector of $\angle AMC$ intersect AC at E. Finally, let F be the midpoint of DE. Find AF.

10. (10) We place a knight in a 3x3 grid such that it has at least one valid move (a knight, as defined in chess, can move two steps horizontally in either direction and one step vertically in either direction, or two steps vertically and one step horizontally). How many ways can we make 16 moves with it such that it ends up in the same square it starts on?